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Dans cette communication, l'auteur démontre la valeur de l'application de la distribution de probabilité hypergéométrique au problème de l'évaluation de la validité d'un test composé d'items pris au hasard dans une banque d'items. Il présente une façon adéquate d'évaluer la variation possible de l'erreur de mesure et d'éviter ainsi la construction d'un test avec plus d'items qu'il n'est nécessaire ou, ce qui est pire, la construction d'un test avec trop peu d'items. De plus, quand on dispose d'un grand nombre d'items pour mesurer une caractéristique quelconque, il est avantageux d'utiliser un échantillon d'items pris au hasard, mais sans accroître pour autant l'erreur de mesure qui n'est pas proportionnelle à la taille de la banque.

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APPLICATION OF THE HYPERGEOMETRIC PROBABILITY DISTRIBUTION TO THE PROBLEM OF ESTIMATING THE RELIABILITY OF A TEST COMPOSED OF ITEMS RANDOMLY SELECTED.

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APPLICATION OF THE HYPERGEOMETRIC PROBABILITY DISTRIBUTION TO THE PROBLEM OF ESTIMATING THE RELIABILITY OF A TEST COMPOSED OF ITEMS RANDOMLY SELECTED

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ABSTRACT

This paper shows how and when the hypergeometric probability distribution is adequate when dealing with the error of measurement in the context of item banking. It shows how and when the binomial probability distribution is not a good approximation of the hypergeometric one when considering an item bank problem. Two indexes of reliability are suggested. These indexes could be used when the items are randomly selected from a bank; they take into account the size of the bank, the size of the test and the level of mastery of a specific individual. This paper also looks over the relation between the level of mastery and the size of the error of measurement in the context of item banking, insisting on the fact that homogeneous banks tend to give slightly higher reliability indexes. The paper also presents a fortran program that is centred on the hypergeometric probability distribution and that is suited to the specific way of defining the error of measurement. (Author)

Suggested descriptors: ITEM BANKS / RELIABILITY / ERROR OF MEASUREMENT

INTRODUCTION

One can see the selection of the items of a test in three different ways. First, the items can be seen as having been chosen one by one on their own merits and not because they are representative of something. This is often the case when a teacher makes a test. Second, the items of a test can be seen as having been randomly selected from an infinite number of items which they represent; this is the theoretical model often adopted by psychometrists. Finally, the third possibility is to see the items of a test as having been randomly selected from a finite set of items, or for the purposes of the discussion here, from an item bank.

This third situation is the one which interests us here since teachers and other test makers are becoming increasingly concerned with the construction and improvement of item banks. This practice becomes more and more feasible with the implementation of computers in the classroom and seems particularly convenient in a context where it appears relevant and even necessary to give students the opportunity to take a test again which refers to the same content but uses different items.

In our paper, we demonstrate the value of an application of the hypergeometric probability distribution to the problem of estimating the reliability of a test composed of items randomly selected from a bank of items.

The subject of such a paper is relevant because the knowledge of a good way to estimate the possible variation of the error of measurement can help in avoiding the construction of a test with more items than it need have or, what is worse, a test with too few items considering the size of the bank. Furthermore, when a large number of items is available for measuring some trait or characteristic, one may take advantage of using a random sample of the items without increasing considerably the error of measurement which is not proportional to the size of the bank.

A SPECIFIC DEFINITION OF THE ERROR OF MEASUREMENT

Our paper is based on a specific definition of the error of measurement.

Before presenting the equation of this error of measurement, let us suppose that:

N is the number of items in the bank;
n is the number of items in the test;
M is the number of items in the bank that are mastered;
m is the number of items in the test that are mastered.

Then we can also define the two following quantities:

$$p = m/n \qquad \text{and} \qquad P = M/N$$

The quantity p represents the proportion of test items that are mastered and P, the proportion of bank items that could be

mastered if they were all present in the test. P can also be seen as the level of mastery of a specific individual.

The error of measurement can be defined as follows:

$$E = |m/n - M/N| = |p - P|$$

Here, the two vertical lines indicate that the sign of the difference between the two proportions is irrelevant, or in other words, that the error refers mainly to the size of the difference.

It is interesting to note that the distribution of such an error of measurement can be estimated. If one supposes that the choice of the items from the bank is completely random, it is possible for any combination of N, n and M, to obtain the probability of occurence for each value that E can take. The hypergeometric probability distribution is well suited to this type of operation; using the terms already defined, the equation for the distribution can be presented as follows:

$$\text{prob.} = \frac{C_m^M \times C_{n-m}^{N-M}}{C_n^N}$$

The entire paper is based on the definition of the error of measurement just given and on the association of this error with an hypergeometric model.

Defining the error of measurement in this way, we consider it as a sampling error. The value of such an error will thus depend on the size of the population that is here the size of the bank (N), on the size of the sample that is here the size of the test (n) and finally, on the proportion of the elements in the population that

have a specific characteristic that is here the level of mastery (P). This model is simple and has all the advantages and all the disadvantages of ignoring any other characteristics associated with the items, as for example, the item difficulty index or any more subtle index. We nevertheless believe that such a model is adequate for most of the testing situations in the real class settings and that more complex models are often unduly sophisticated.

BINOMIAL VS HYPERGEOMETRIC PROBABILITY DISTRIBUTION

After having accepted the error of measurement as a sampling error, one could be inclined to consider that the binomial probability distribution is well suited this situation.

Let us recall, however, that the binomial probability distribution is a good approximation of the hypergeometric probability distribution only when the ratio n/N is very small; in fact, when this ratio approaches 0,, the two distributions are nearly equivalent. The binomial could be a good approximation if the item of a test could be seen as randomly chosen from a bank containing a very large number of items or even better, from an infinite number of items. This assumption seems to us as being inadequate in most of the cases of tests constructed from an item bank.

Table I shows, for nine combinations of N and n, the ratio of the standard deviations when the binomial is considered and when the hypergeometric probability distribution is considered; note that a value of 0,50 is considered for the computations of standard deviations. The nine combinations of N and n selected here are good examples, we think, of typical situations for tests constructed from

TABLE 1

Comparison of the standard deviation of the binomial probability distribution and of the hypergeometric probability distribution for nine combinations of N and n

size of the bank	size of the test	σ_B* (binomial probability distribution)	σ_H (hypergeometric probability distribution)	σ_H / σ_B
20	10	1,5811	1,1471	0,73
30	16	2,000	1,3896	0,69
50	20	2,2361	1,7496	0,78
50	30	2,7386	1,7496	0,64
80	30	2,7386	2,1787	0,80
80	40	3,1623	2,2502	0,71
100	40	3,1623	2,4618	0,78
100	50	3,5355	2,5126	0,71
200	50	3,5355	3,0695	0,87

* For the above table, the value of P that is considered is 0,50. Note however that the ratios in the last column would be the same for whatever value of P that could be considered.

an item bank. The ratios in the table show that the standard deviation is not only always smaller when the hypergeometric is considered, but also show that the differences between the standard deviations are large enough to be considered in the context of error of measurement.

So, we consider that in many situations the hypergeometric probability distribution is more adequate than the binomial probability distribution when dealing with the error of measurement in the context of item banking.

PROPOSITION OF A SPECIFIC INDEX OF RELIABILITY

The size of the error of measurement depends on the value of N, n and P and in a certain way, the standard deviation of the hypergeometric probability distribution gives an idea of the reliability of a score when a specific combination of N, n and P is considered.

The standard deviation, however, proves to have in this context some disadvantages. First, the interpretation cannot well take into account some characteristics of the probability distribution, namely its skewness and its kurtosis. Second, it is useful to have an index of reliability that is comparable from one situation to another taking into account the size of the test. But above all, we thought that a more informative index of reliability was needed.

We propose here the idea of an index of reliability or to be more exact, a type of index of reliability.

The idea is to compute for a specific combination of N, n and P, the percentage of chances that one has of not observing a difference between p and P larger than a specific range. This range could well be 0,05 or 0,10 for example; the index of reliability would then reflect the probability of not observing an error of measurement larger than 5 percent or 10 percent. Such statistics are quite simple and correspond fairly well, we think, to the kind of information that should be expected by test users in terms of reliability.

Table 2 shows, for twenty-seven combinations of N, n and P, an index of reliability when considering a range of 5 percent and then when considering a range of 10 percent.

The first line of the table can be read as follows: if the size of the bank is 20, the size of the test 10, and the level of mastery for a specific individual 0,50, the probability of observing an error of more than 5 percent when using a random test is 0,3437 and the probability of having an error of more than 10 percent is 0,8211.

In order to use such probabilities, one has to assume a level of mastery for a specific individual. Theoretically, this problem is insolvable since P is by definition unknown; let us recall that we are precisely concerned with the estimation of the quantity P. An estimate of P has to be considered and as it is current in statistics, the best estimate of P here should be p, that is the level of mastery of the individual as measured by the test.

It is however important to note that the problem of having to determine the hypothetical level of mastery of a specific individual is not as important as it first looks, since one can always assume the level of mastery that leads to the maximum error of

TABLE 2

The two suggested indexes of reliability for twenty-seven combinations of N, n and P

size of the bank	size of the test	level of mastery	probability (error < 5%)	probability (error < 10%)
020	010	0,50	0,3437	0,8211
020	010	0,70	0,3715	0,8591
020	010	0,80	0,4180	0,9133
030	016	0,50	0,2848	0,7277
030	016	0,70	0,5603	0,7640
030	016	0,80	0,6222	0,8245
050	020	0,50	0,6132	0,8518
050	020	0,70	0,6549	0,8864
050	020	0,80	0,7211	0,9327
050	030	0,50	0,6132	0,9579
050	030	0,70	0,6549	0,9745
050	030	0,80	0,7211	0,9912
080	030	0,50	0,5112	0,8947
080	030	0,70	0,5498	0,9236
080	030	0,80	0,6129	0,9593
080	040	0,50	0,7365	0,9565
080	040	0,70	0,7778	0,9730
080	040	0,80	0,8390	0,9897
100	040	0,50	0,6926	0,9344
100	040	0,70	0,7347	0,9561
100	040	0,80	0,7988	0,9800
100	050	0,50	0,6827	0,9727
100	050	0,70	0,7248	0,9844
100	050	0,80	0,7893	0,9950
200	050	0,50	0,5856	0,9282
200	050	0,70	0,6269	0,9510
200	050	0,80	0,6929	0,9766

measurement, that is 0,50. In fact, the standard deviation of the hypergeometric probability distribution is at its maximum when P is equal to 0,50. So, when assuming a level of mastery of 50 percent, one can be sure that the estimate of the mean value of the error of measurement is necessarily an overestimation, as far as sampling is concerned.

DETERMINING A STANDARD FOR MASTERY

Let us consider now the problem of determining a mastery standard for a random test. This point is of particular interest to those who are concerned with issues relating to criterion-referenced tests.

In the present context, we define the mastery standard as the minimum score necessary in order to consider that there is success or that the objective has been reached. We deal here with the problem of deciding if a standard for mastery is reached when considering a sample of items. The standard of mastery is associated with the set of items in the bank and not to the score on the test itself; that is why we once again have to deal with an estimation problem.

Two kinds of wrong decisions can be made. The first kind of wrong decision is to decide that there is success when the standard for mastery would not be reached at the level of the bank. For example, if the observed score of a student is 83 when the standard for mastery is 80 and when the true level of mastery (P) of that student is only 75. Second, there is the opposite situation, that is when the observed score is too low compared to the true score:

then one could conclude that the standard for mastery is not reached when that standard would be reached at the level of the bank. These two decisions are wrong but one is really worse than the other as far as pedagogical considerations are concerned. In some ways, to underestimate the performance of a student accidentally has less consequences on the progress of the student than if one considered that he has a good level of mastery when it is not the actual situation.

However, here we limit our attention to the overestimation of the performance of a student when using a random test. The other kind of error is ignored here.

For a given standard of mastery, we propose computing the percentage of chances of observing a score that is equal or higher to that standard when the value of P (in some ways, the true score) is 5 ou 10 points lower than the standard of mastery. Table 3 gives that kind of information for seven combinations of N and n when a fixed standard of mastery of 80% is considered.

The first line of the table can be read as follows. It indicates that the size of the bank is 50 when the size of the test is 20. This first line assumes a hypothetical level of mastery (P) that is as near as possible to 75% (that is 80% minus 5%); it corresponds to a possibility of 37 good answers out of 50. The value of 0,3263 is the probability that if the true performance of a specific individual is 74%, a score equal to or higher than 80% be observed. For that specific combination, the chances of a wrong decision are fairly hygh. The following line of the table shows the same probability when a hypothetical level of mastery of 70% is considered. It is evidently higher than the previous one.

TABLE 3

Probability of observing a score higher than or equal to 80% for two hypothetical levels of mastery and for seven combinations of N and n

size of the bank	size of the test	hypothetical level of mastery		probability (observed score ≥ 80%)
		%	n. of good answers	
50	20	74	37	0,3263
		70	35	0,1729
50	30	74	37	0,1956
		70	35	0,0583
80	30	75	60	0,3002
		70	56	0,1027
80	40	75	60	0,2196
		70	56	0,0433
100	40	75	75	0,2413
		70	70	0,0580
100	50	75	75	0,1779
		70	70	0,0243
200	50	75	150	0,2276
		70	140	0,0518

THE RELATION BETWEEN THE LEVEL OF MASTERY OF A SPECIFIC INDIVIDUAL AND THE INDEX OF RELIABILITY

In the field of achievement measurement, we are too used to considering that interesting items are those having a level of difficulty that approaches 0,50; such items are seen, all things being equal, as contributing more to the variance of the test, and in a specific context, to a higher estimate of reliability.

For the problem we are now considering, things are different. In fact, one could observe that the index of reliability is at its lowest point when the level of mastery is equal to 50 percent; the reliability, or to be more exact, the index of reliability tends towards 1 when the level of mastery approaches 0 or 100 percent.

In others words, in a general way, the error of measurement tends to decrease as much as the level of mastery differs from 0,50. This situation favorizes the idea of having a standard of mastery that is high, for example, nearer to 80 percent than to 50 percent.

Up to now, in the present paper, we have insisted on the relation between the level of mastery of a specific individual and on the reliability. This relation can however also be seen in terms of a relation between the reliability and the homogeneity of the bank. If one can suppose that the items of a bank are concentrated on a same field, a same domain, or a same objective, the greater are the chances that the level of mastery of a student will go away from 50 percent; the more homogeneous the item bank considered is, the more a specific student will tend to have a good answer for most of the items or, in the opposite situation, for hardly any of the items. The idea of homogeneity of the bank refers to a level of mastery that goes away from 50 percent.

In terms of reliability as defined here, chances for having small errors of measurement depend partly on the degree of homogeneity of the bank. However, as table 2 indicated, the relation is not strong enough to favorize homogeneous banks; if one goes in that direction, it should be mainly for pedagogical considerations, namely in order to favorize a better interpretation of test results more than to increase the reliability of these results.

THE PRODUCTION OF A SET OF TABLES FOR TEST USERS

The present paper gives only a few probabilities refering only to a small number of combinations of N and n. We thought it could be useful for test users to dispose of a set of tables containing more information for many combinations of N, n and P.

In order to get the statistics that we consider helpful a lot of calculations would be necessary. And even the most complete hypergeometric probability distribution tables deal only with an incomplete number of combination of N, n and P.

That is why we decided to produce a set of particular tables for more than 150 combinations of N, n and p that were considered as being the most relevant in the context of item banking.

Such tables can be produced by using a fortran program that we previously developed in the context of our work on item banking.

This program, that is presented as an appendix of this paper, needs as a starting point only three parameters: the size of the bank, the size of the test and the level of mastery.

We present in tables 4 and 5 an output example of the program. Table 4 refers to the first part of the output and table 5 to the second. This example considers that the size of the bank is 200, the size of the test is 50 and that the level of mastery is 70%.

Table 4 is composed as follows. The first column gives the scores that is the possible number of good answers. The second column gives the probability of occurence of each score. The third column gives the percentage; in fact, it is the score divided by the number of items in the test. The forth column indicates the value of the error of measurement for each score. Finally, the fifth and sixth columns give the cumulative probability; these two last columns are, in fact, related to the second column.

At the bottom of the table, the standard deviation for the score and the percentage, as well as, the mean and the standard deviation for the error are given. Finally, the table gives two indexes of reliability as defined before, one for an error of less than 5% and one for an error of less than 10%.

Table 5 refers to the second part of the output of the program. Such a table exists for each combination of N and n. The first column presents the different standards for mastery that are considered by the program; the number in parenthesis indicates the number of good answers that correspond to the fixed standards of mastery; it is this number that is considered in the calculations. The second column presents the hypothetical level of mastery and the nearest integer number is in parenthesis. Finally, the third column gives the probability of occurence of each event.

The first line of table 5 can be read as follows: it indicates the probability of observing a number of good answers equal to or

TABLE 4

First part of the output of the fortran program presented in this paper

SIZE OF THE BANK : 200
SIZE OF THE TEST : 50
N. OF ITEMS OF THE BANK THAT ARE MASTERED : 140
LEVEL OF MASTERY : 70.00%

SCORE	PROB.	%	ERROR	CUM. PROB. (POSITIVE)	CUM. PROB. (NEGATIVE)
43	.0020	86.00	16.00	.0027	.9973
42	.0057	84.00	14.00	.0084	.9916
41	.0140	82.00	12.00	.0225	.9775
40	.0294	80.00	10.00	.0518	.9482
39	.0529	78.00	8.00	.1047	.8953
38	.0825	76.00	6.00	.1872	.8128
37	.1124	74.00	4.00	.2997	.7003
36	.1343	72.00	2.00	.4340	.5660
35	.1412	70.00	0.00	.5752	.4248
34	.1311	68.00	-2.00	.7063	.2937
33	.1078	66.00	-4.00	.8141	.1859
32	.0787	64.00	-6.00	.8929	.1071
31	.0511	62.00	-8.00	.9440	.0560
30	.0295	60.00	-10.00	.9735	.0265
29	.0152	58.00	-12.00	.9887	.0113
28	.0070	56.00	-14.00	.9956	.0044
27	.0029	54.00	-16.00	.9985	.0015

SCORE (STANDARD DEVIATION) : 2.81
% (STANDARD DEVIATION) : 5.63

ERROR (MEAN) : 29.41 %
ERROR (STANDARD DEVIATION) : 20.04 %

PROB. (ERROR< 5%) : .6269
PROB. (ERROR< 10%) : .9510

TABLE 5

Second part of the output of the fortran program presented in this paper

SIZE OF THE BANK :			200
SIZE OF THE TEST :			50
EVENT	HYPOTHETICAL LEVEL OF MASTERY		PROB.
90% (45)	85%	(170)	.1815
	80%	(160)	.0285
	75%	(150)	.0027
84% (42)	80%	(160)	.2748
	75%	(150)	.0625
	70%	(140)	.0084
80% (40)	75%	(150)	.2276
	70%	(140)	.0518
	65%	(130)	.0070
74% (37)	70%	(140)	.2997
	65%	(130)	.0840
	60%	(120)	.0140
70% (35)	65%	(130)	.2484
	60%	(120)	.0656
	55%	(110)	.0102
64% (32)	60%	(120)	.3102
	55%	(110)	.0941
	50%	(100)	.0166
60% (30)	55%	(110)	.2564
	50%	(100)	.0706
	45%	(90)	.0109

higher than 45 (90%) when the level of mastery of a specific individual refers to a possibility of 170 good answers out of 200 (85%); this probability is equal to 0,1815.

Table 5 gives this kind of information for seven fixed standards for mastery and three ranges for each one (5%, 10% and 15%).

CONCLUSION

We have tried to show how and when the hypergeometric probability distribution is adequate when dealing with the error of measurement in the context of item banking. We have shown how and when the binomial probability distribution is not a good approximation of the hypergeometric one when considering an item banking problem. Two indexes of reliability have been suggested. These indexes could be used when the items of a test are randomly selected from a bank; they take into account the values of N, n and M.

This paper also looks over the relation between the value of M and the size of the error of measurement, insisting on the fact that homogeneous banks tend to give slightly higher reliability indexes. The problem of the determination of a mastery standard for a random test is also approached.

This paper terminates with the presentation of a fortran program that has been built in order to give many statistics related to the error of measurement for any combination of N, n and M.

REFERENCES

Freeman, P.R. (1973). Algorithm AS 59. Hypergeometric probabilities. Appliec Statistics, 22, 130-133.

Hsu, Louis M., Determination of the number of items and passing score in a mastery test. Educational and Psychological Measurement, 1980, volume 40, number 7, 709-714.

Lord, Frederic M. and Novick, Melvin R., Statistical Theories of Mental Test Scores. 1968, Addison-Wesley Publishing Company.

Larson, Harold, J., Introduction to Probability Theory and Statistical Inference. 1969, John-Wesley and Sons Inc.

Lieberman Gerald J., Owen Donald B. Tables of the Hypergeometric Probability Distribution, 1961. Standford University Press.

Lund, R.F. (1980). Algorithm AS 152. Cumulative hypergeometric probabilities. Applied Statistics, 29, 221-223.

APPENDIX

The fortran program presented here has been developed by Normand Dufour who is a research assistant at INRS-Education. This program is initially based on another fortran program dealing with the hypergeometric probability distribution and presented in Applied Statistics, volume 29, 221-223.

```
      PROGRAM PBANK(INPUT,OUTPUT)
      DIMENSION PRER(2),HYP(3),VER(2)
      INTEGER ISL(3),ITEC(3),IVER(3)
C
CCC   NN : SIZE OF THE BANK
CCC   N  : SIZE OF THE TEST
CCC   LV : LEVEL OF MASTERY
C
      READ 100,NN,N,LV
      RP=FLOAT(LV)/100.0
      MM=NN*RP+0.5
      RP=RP*100.0
      CUM=0
      CER=0
      SER=0
  100 FORMAT(3I3)
      IF (NN.GE.MM) GOTO 120
      PRINT 105,MM,NN
  105 FORMAT(3(/),5X,*N. OF ITEMS OF THE BANK THAT ARE MASTERED (*,
     1I3,*) > SIZE OF THE BANK (*,I3,*)*)
      GO TO 77
  120 IF (NN.GE.N) GOTO 130
      PRINT 115,N,NN
  115 FORMAT(3(/),5X,*SIZE OF THE TEST (*,I3,*) > SIZE OF *
     1,*BANK (*,I3,*)*)
      GO TO 77
C
CC    PART 1
C
  130 PRINT 1000
 1000 FORMAT(1H1)
      PRINT 200,NN,N,MM,RP
  200 FORMAT(3(/),9X,*SIZE OF THE BANK !*,26X,I3,/,9X,
     1*SIZE OF THE TEST !*,26X,I3,/,9X,*N. OF ITEMS OF THE BANK *,
     2*THAT ARE MASTERED !*,I4,/,9X,*LEVEL OF MASTERY :*,25X,
     3F6.2,*%*,3(/))
      PRINT 600
      DO 11 I=1,2
      PRER(I)=0
      VER(I)=5.*I+0.005
   11 IVER(I)=5*I
      M1=N+1
      DO 1 IRR=1,M1
      IR=N-IRR+1
      HYPP=HYPER(1,N,IR,NN,MM,IFAULT)
      CUM=CUM+HYPP
      CUMN=1-CUM
      RS=(FLOAT(IR)/FLOAT(N))*100
      ERR=RS-RP
      CER=CER+ERR**2
      SER=SER+ABS(ERR)
      IF((CUM.LT.0.001).OR.(CUMN.LT.0.001)) GO  TO 1
      DO 2 I=1,2
      IF (ABS(ERR).GE.VER(I)) GO TO 2
      PRER(I)=PRER(I)+HYPP
    2 CONTINUE
      PRINT 300,IR,HYPP,RS,ERR,CUM,CUMN
    1 CONTINUE
      PRINT 999
  999 FORMAT(2X,63(1H*))
```

```
      A1=RP/100
      ETRS=SQRT((NN-N)*N*A1*(1-A1)/(NN-1))
      ETER=SQRT(M1*CER-SER**2)/M1
      ECM=SER/M1
      ETPR=100*ETRS/N
      PRINT 400,ETRS,ETPR,ECM,ETER
  400 FORMAT(4(/),9X,*SCORE (STANDARD DEVIATION) :*,5X,F12.2,/,9X,*%*,
     15X,*(STANDARD DEVIATION) :*,5X,F12.2,2(/),9X,*ERROR (MEAN)*,
     215X,*:*,5X,F12.2,* %*,/,9X,*ERROR (STANDARD DEVIATION) :*,
     35X,F12.2,* %*)
      PRINT 500,(IVER(I),PRER(I),I=1,2)
  500 FORMAT(7(/),2(9X,*PROB.(ERROR<*,I3,*%) :*,22X,F6.4,/))
  600 FORMAT(3(/),2X,63(1H*),/,2X,1H*,7X,1H*,3(9X,1H*),2(2X,*CUM
     15X,1H*),/,2X,1H*,8H SCORE *,2X,*PROB.*,3H  *,4X,*%*,4X,1H*,
     22X,*ERROR*,3H  *,2(2X,*PROB.*,4X,1H*),/,2X,1H*,7X,1H*,3(9X,1H*),
     31X,*(POSITIVE)*,1H*,1X,*(NEGATIVE)*,1H*,/,2X,63(1H*),
     4',2X,1H*,7X,1H*,3(9X,1H*),2(11X,1H*))
  300 FORMAT(2X,1H*,I5,2X,1H*,F7.4,2X,1H*,F7.2,2X,1H*,F8.2,1X,1H*,
     12(F8.4,3X,1H*))
 2000 FORMAT(11(/))
      IF ((NN.LT.20).OR.(N.LT.20)) GO TO 77
C
CC    PART 2
C
      PRINT 1000
      PRINT 2000
      PRINT 900,NN,N
  900 FORMAT(3(/),15X,*SIZE OF THE BANK :*,26X,I3,/,15X,
     1*SIZE OF THE TEST :*,26X,I3,7(/))
      PRINT 700
      DO 6 LN=1,7
      SEUIL=0.90-((LN-1)*0.05)
      ITB=INT(N*SEUIL+0.5)
      X=ITB/FLOAT(N)+0.005
      ICS=INT(100*X)
      DO 7 II=1,3
      X=SEUIL-II*0.05
      ITEC(II)=INT(NN*X+0.5)
      X=ITEC(II)/FLOAT(NN)+0.005
      ISL(II)=INT(100*X)
    7 HYP(II)=HYPER(2,N,N-ITB,NN,NN-ITEC(II),IFAULT)
      PRINT 800,ICS,ITB,(ISL(II),ITEC(II),HYP(II),II=1,3)
    6 CONTINUE
  700 FORMAT(5(/),15X,*EVENT*,11X,*HYPOTHETICAL LEVEL*,12X,*PROB.*/,
     135X,*OF MASTERY*,2(/))
  800 FORMAT(12X,I3,*%(*,I3,*)*,12X,I3,*%*,4X,*(*,I3,*)*,14X,F6.4,/,
     12(33X,I3,*%*,4X,*(*,I3,*)*,14X,F6.4,/))
   77 STOP
      END
      FUNCTION HYPER(MODE,KK,LL,MM,NN,IFAULT)
      DATA ZERO/0.0/,ONE/1.0/
C
CC    MODE .EQ. 1 : HYPERGEOMETRIC PROBABILITY
CC
CC    MODE .NE. 1 : CUMULATIVE HYPERGEOMETRIC PROBABILITY
C
      K=KK+1
      L=LL+1
      M=MM+1
      N=NN+1
      IDIR=1
      IFAULT=1
```

```
C
C     CHECK ARGUMENTS
C
      IF(N.LT.1.OR.M.LT.N.OR.K.LT.1.OR.K.GT.M) RETURN
      IFAULT=2
      HYPER=ZERO
      IF(L.LT.1.OR.K-L.GT.M-N) RETURN
      IF(MODE.NE.1) HYPER=ONE
      IF(L.GT.N.OR.L.GT.K) RETURN
      IFAULT=0
      HYPER=ONE
      IF(K.EQ.1.OR.K.EQ.M.OR.N.EQ.1.OR.N.EQ.M)  RETURN
C
C     INTERCHANGE K AND N IF THIS SAVES CALCULATIONS
C
      IF(MINO(K-1,M-K).LE.MINO(N-1,M-N)) GO TO 1
      I=K
      K=N
      N=I
    1 IF(M-K.GE.K-1) GO TO 2
      IDIR=-IDIR
      L=N-L+1
      K=M-K+1
    2 IF(L.EQ.1) GO TO 4
      LA=L-1
      DO 3 I=1,LA
    3 HYPER=HYPER*FLOAT((K-I)*(N-I))/FLOAT((L-I)*(M-I))
      IF(L.EQ.K) GO TO 6
    4 J=M-N+L
      LA=K-1
      DO 5 I=L,LA
    5 HYPER=HYPER*FLOAT(J-I)/FLOAT(M-I)
    6 IF(MODE.EQ.1) RETURN
C
C     CUMULATIVE PROBALITY
C
      P=HYPER
      PT=ZERO
      NL=N-L
      KL=K-L
      MNKL=M-N-KL+1
      IF (L.GT.KL) GO TO 8
      IF (L.EQ.1) GO TO 10
      LA=L-1
      DO 7 I=1,LA
      P=P*FLOAT((L-I)*(MNKL-I))/FLOAT((NL+I)*(KL+I))
      PT=PT+P
    7 CONTINUE
      GO TO 10
    8 IDIR=-IDIR
      IF (KL.EQ.0) GO TO 10
      DO 9 I=1,KL
      J=I-1
      P=P*FLOAT((NL-J)*(KL-J))/FLOAT((L+J)*(MNKL+J))
      PT=PT+P
    9 CONTINUE
   10 IF (IDIR.LT.0) GO TO 11
      HYPER=HYPER+PT
      RETURN
   11 HYPER=ONE-PT
      RETURN
      END
```